A NEW ALGORITHM FOR PHASOR ESTIMATION IN UNBALANCED THREE-PHASE POWER SYSTEMS

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Keywords: *Complex Least Mean Square (CLMS), frequency estimation, unbalanced three-phase voltage, balanced transformation.*

Abstract: A new iterative method for phasor estimation of power system is proposed. The proposed technique uses three adaptive algorithms. First two algorithms are used for frequency estimation. Third adaptive algorithm estimates amplitudes and phase angles between phasors. Simulation results confirm that the proposed algorithm exhibits better performances compared to the considered ones.

1. INTRODUCTION

The conventional phasor estimation techniques in power systems are based on the assumption of constant frequency. Since the frequency is dynamic, its real-time estimation is very important in order to avoid the errors in phasor modeling, [1]. Also, the frequency fluctuations reflect a balance between power generation and load consumption, [1]. Therefore, the frequency is an important parameter in power system monitoring, control and protection [1, 2].

Techniques for frequency estimation are usually based on using the digitalized samples of the voltage signal. Considering that the system voltage is a purely sinusoid, the time between two zero crossings gives information about the system frequency, [3, 4]. However, the measured data is usually noised and, therefore, many iterative methods for frequency estimation have been proposed. Discrete Fourier transform (DFT), Kalman filtering, Recursive Least Square (RLS) and various iterative methods are some of the proposed approaches, [5-12]. Soft computing technique, such as genetic algorithms and neural networks, are also used in power system frequency estimation, [13, 14]. The application of the Viterbi algorithm in frequency estimation is presented in [15].

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Most of the mentioned techniques are designed for single-phase system, which is not enough for characterization of three-phase power system. The optimal method for frequency estimation should be based on using the information about all the three system phases, in order to achieve a high robustness, [21]. For this reason, the Clarke's *αβ* transform, which produce a complex signal by considering the information among all the three phases, is introduced, [16]. A variety of techniques for frequency estimation based on the *αβ*-transformed signal are proposed, [17-20].

In this paper a new real-time method for phasor estimation in unbalanced power systems is proposed. The proposed solution uses three adaptive algorithms. The first adaptive algorithm (Newton LMS) searches for optimal coefficients which transform the *αβ* complex signal into a complex exponential. At the same time, based on the transformed signal, the second adaptive algorithm (the Complex LMS, CLMS) is used for power system frequency estimation. The third adaptive algorithm (LMS) estimates the amplitudes and phase angles of three-phase voltages by using the coefficients obtained by Newton LMS.

The paper is organized as follows. A brief description of the CLMS and WLCLMS is given in Section 2. In Section 3 the proposed algorithm for phasor estimation is presented. Finally, the simulation results and concluding remarks are given in Sections 4 and 5.

2. FREQUENCY ESTIMATION USING CLMS ALGORITHM

The three-phase voltages in a balanced power system can be represented in a discrete form as:

$$\begin{aligned} v_a(n) &= V\cos(\omega nT + \phi) + \eta_a(n) \\ v_b(n) &= V\cos(\omega nT + \phi - 2\pi/3) + \eta_b(n) \\ v_c(n) &= V\cos(\omega T + \phi + 2\pi/3) + \eta_c(n), \end{aligned} \tag{1}$$

where V is the peak value of the fundamental voltage component, T is the sampling interval, ϕ is the initial phase angle, ω is the angular frequency of the voltage signal (angular frequency of the voltage signal (ω=2πf , with f being the system frequency), whereas $\eta_a(n)$, $\eta_b(n)$ and $\eta_c(n)$ denote the additive noise of the respective phases. We assume that the noises are zero-mean i.i.d.

Application of the Clarke's transform to the noisy three-phase voltages gives, [21]:

$$\begin{bmatrix} v_\alpha(n) \\ v_\beta(n) \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & -\frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} v_a(n) \\ v_b(n) \\ v_c(n) \end{bmatrix}. \tag{2}$$

Combining $v_\alpha(n)$ and $v_\beta(n)$ results in a complex representation of the three-phase power system:

$$v(n) = v_{\alpha}(n) + jv_{\beta}(n), \tag{3}$$

Equation (3) can be written as follows:

$$v(n) = Ve^{j(\omega nT+\phi)} + \eta(n) = \hat{v}(n) + \eta(n). \tag{4}$$

The voltage $\hat{v}(n)$ can be modeled as:

$$\hat{v}(n) = Ve^{j(\omega nT+\phi)} = \hat{v}(n-1)e^{j\omega T}. \tag{5}$$

The model given by (5) is utilized in the CLMS algorithm for frequency estimation, [20]:

$$W(n) = W(n-1) + \mu(n)e(n)\hat{v}^{*}(n), \tag{6}$$

where * denotes a complex conjugate operation, $\mu(n)$ is the step-size which controls the convergence speed and stability of the algorithm, $e(n)$ is the error signal defined as:

$$e(n) = v(n) - \hat{v}(n), \tag{7}$$

whereas $W(n)$ is the adaptive coefficient which converges to $e^{j\omega T}$. At each time instant n, the frequency is calculated using, [20]:

$$\hat{f}(n) = \frac{1}{2\pi T}\sin^{-1}[\mathrm{Im}(W(n)], \tag{8}$$

where Im represents the imaginary part of a complex-value number.

Generally, three-phase voltages may have arbitrary amplitudes and initial phases:

$$\begin{aligned} v_a(n) &= V_a\cos(\omega nT + \phi) + \eta_a(n) \\ v_b(n) &= V_b\cos(\omega nT + \phi + \varphi_b) + \eta_b(n) \\ v_c(n) &= V_c\cos(\omega T + \phi + \varphi_c) + \eta_c(n), \end{aligned} \tag{9}$$

where V_a, V_b and V_c are the peak values of each fundamental component, φ_b and φ_c are respectively phase angles of phases v_b and v_c relative to the phase v_c.

In the unbalanced systems, the $\alpha\beta$-transformed complex voltage signal can be written as:

$$\begin{aligned} v(n) = {} & Ae^{j\omega nT} + A^{*}e^{-j\omega nT} + \\ & jBe^{j\omega nT} + jB^{*}e^{-j\omega nT} + \eta(n), \end{aligned} \tag{10}$$

where *A* and *B* are equal:

$$A = \frac{e^{j\phi}}{3}\left(V_a - \frac{V_b e^{j\varphi_b}}{2} - \frac{V_c e^{j\varphi_c}}{2}\right)$$
$$B = \frac{e^{j\phi}}{2\sqrt{3}}\left(V_b e^{j\varphi_b} - V_c e^{j\varphi_c}\right). \tag{11}$$

The CLMS-based estimator exhibits a good performance in balanced power system conditions, i.e. when voltage signals are given by (1). In [21] it is shown that for unbalanced conditions the CLMS algorithm is not capable of modeling the complex voltage. Namely, in the steady state, the adaptive coefficient $W(n)$ oscillates at twice of the system frequency:

$$W(n) = W(n + \frac{1}{2f\Delta T}). \tag{12}$$

In order to accurately estimate the frequency in unbalanced power system conditions, the Widely Linear CLMS (WLCLMS) algorithm is proposed, [21]. The complex voltage signal is modeled in the following way, [21]:

$$\hat{v}(n) = v(n-1)h(n) + v(n-1)^* g(n), \tag{13}$$

where $v(n)$ is the complex signal given by (10), $h(n)$ and $g(n)$ are the complex adaptive coefficients that are updated in the following way:

$$h(n+1) = h(n) + \mu e(n) v^*(n),$$
$$g(n+1) = g(n) + \mu e(n) v(n). \tag{14}$$

The error signal $e(n)$ is defined in the same way as in the CLMS algorithm.

At each iteration the frequency is updated using, [21]:

$$\hat{f}(n) = \frac{1}{2\pi T}\arctan\frac{\mathrm{Im}\{h(n)\} + a_1(n)g(n)}{R\{h(n) + a_1(n)g(n)\}}, \tag{15}$$

where:

$$a_1(n) = \frac{-j\,\mathrm{Im}\{h(n) + j\sqrt{\mathrm{Im}^2\{g(n)\} - |g(n)|^2}}{g(n)}. \tag{16}$$

3. PROPOSED ALGORITHM FOR FHASOR ESTIMATION

The coefficients of $\alpha\beta$ transform for transformation of balanced three-phase system into a complex form are obtained according to the phasor diagram shown in Fig. 1.a.

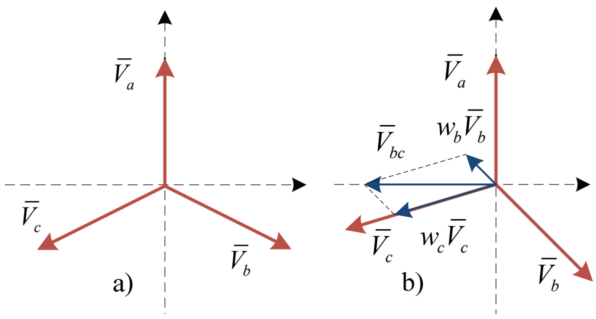

Fig 1. Phasor diagram of a) balanced,
b) unbalanced three-phase power system

From Fig. 1.b it can be observed that always exist the coefficients w_b and w_c that can be chosen in such a way that the vector $\overline{V}_{bc} = w_a\overline{V}_b + w_b\overline{V}_c$ has the same amplitude as the vector $\overline{V}_a$, and that is normal to the vector $\overline{V}_a$. If the coefficients w_b and w_c are known, the complex three-phase voltage can be obtained in the following way:

$$\begin{aligned} V(n) &= V_a(n) + jV_{bc}(n) \\ &= V_a(n) + j(w_bV_b(n) + w_cV_c(n)), \end{aligned} \tag{17}$$

If the amplitude of $v_a(n)$ is known, the coefficients w_b and w_c can be found by minimizing the cost function:

$$J = \varepsilon(n)^2 = \left(V_a^2 - v_a(n)^2 - \left(\begin{bmatrix} w_a & w_b \end{bmatrix} \begin{bmatrix} y_b(n) \\ y_c(n) \end{bmatrix} \right)^2 \right)^2 . \tag{18}$$

The optimal coefficients can be found iteratively by using the Newton LMS (NLMS) algorithm:

$$\begin{bmatrix} w_b(n+1) \\ w_c(n+1) \end{bmatrix} = \begin{bmatrix} w_b(n) \\ w_c(n) \end{bmatrix} + \mu_1\varepsilon(n)R^{-1} \begin{bmatrix} y_b(n) \\ y_c(n) \end{bmatrix}, \tag{19}$$

where μ_1 denotes the algorithm step size, and **R** denotes autocorrelation matrix that is also iteratively estimated:

$$R(n) = \lambda R(n) + (1-\lambda)\begin{bmatrix} y_a(n) \\ y_b(n) \end{bmatrix}\begin{bmatrix} y_a(n) & y_b(n) \end{bmatrix}. \tag{20}$$

The forgetting factor λ should be chosen in the range 0-1.

The CLMS algorithm for frequency estimation is executed parallel with the NLMS algorithm, by using (6), (7) and (8).

In the case when amplitude V_a is unknown, it can be also estimated iteratively:

$$w_a(n+1) = w_a(n) + \mu_2 \varepsilon(n). \tag{21}$$

Beside the frequency, the voltage amplitudes and phase angles should be estimated. Amplitude V_a can be estimated by (21), while V_b, V_c, φ_b and φ_b are uknown.

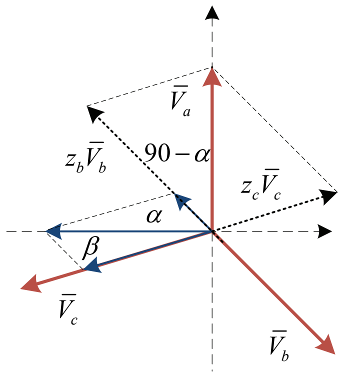

Fig 2. Phasor diagram of unbalanced power system

Fig. 2. shows that always exist the coefficients z_a and z_b such that:

$$\zeta(n) = v_a(n) + z_b v_b(n) + z_c v_c(n). \tag{22}$$

The coefficintns z_a and z_b can be found iteratively, by minimizing the square of the error signal $\zeta(n)$:

$$\begin{aligned} z_b(n+1) &= z_b(n) + \mu_3 \zeta(n) y_b(n) \\ z_c(n+1) &= z_c(n) + \mu_3 \zeta(n) y_c(n) \end{aligned} \tag{23}$$

The other parameters are estimated in the following way:

$$\varphi_b(n) = 90^\circ + \alpha(n),\ \varphi_c(n) = 90^\circ + \beta(n),$$
$$V_b(n) = \frac{w_a}{z_b}\sin\frac{\alpha(n)}{\alpha(n)+\beta(n)}, \tag{24}$$
$$V_c(n) = \frac{w_a(n)}{z_c(n)}\sin\frac{\beta(n)}{\alpha(n)+\beta(n)},$$

where the angles α i β are determined by using the low of sine (Fig. 2):

$$\alpha(n) = \arctan\frac{z_a(n)}{w_a(n)}, \beta(n) = \arctan\frac{z_b(n)}{w_b(n)}. \tag{25}$$

The advantage of the proposed method is that it does not require the information about system frequency for phasor estimation.

4. SIMULATION RESULTS

The performance of the proposed algorithm is verified through numerical simulations. Unbalanced and balanced conditions are considered, as the examples where frequency is constant and time varying. Voltage signals are contaminated with white Gaussian noise, establishing the SNR 40dB. The system frequency is 50Hz, and the sampling frequency has been set at 5 kHz.

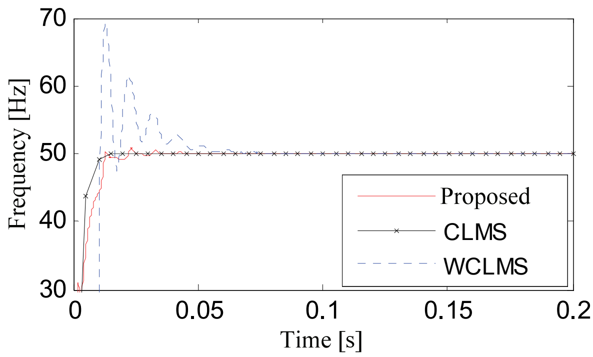

Fig 3. Comparison of the considered algorithms in balanced conditions

In the first example, three-phase voltage is balanced (V_a=V_b=V_c=220V, φ_b=φ_c=120^o). From Fig. 3 it can be seen that the CLMS and proposed algorithm have the similar convergence speed, whereas the WLCLMS converges the slowest.

In the second example, three-phase voltage is unbalanced (V_a=220V, V_b=170V, V_c=180V, φ_b=130^o, φ_c=160^o). From Fig. 4. it can be observed that the proposed algorithm exhibits the fastest convergence speed. The WLCLMS has the slower convergence, whereas the estimated frequency obtained using the CLMS suffers from the oscillation error.

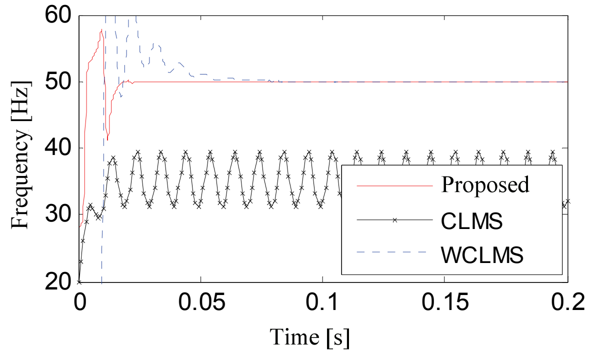

Fig 4. Comparison of the considered algorithms in balanced conditions

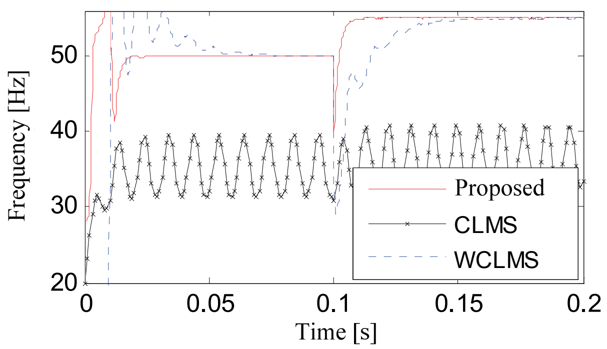

Fig 5. Comparison of the considered algorithms in the case when frequency is time-varying

In the third example the same parameters as in the previous example are used. In t=0.1s the system frequency is changed to 55Hz. Fig. 5 shows the proposed algorithm exhibits the faster convergence compared to the WLCLMS, while the CLMS is not capable of modeling the system frequency.

In Table 1. the estimated phasor parameters are shown. Note that the error in the amplitude estimation and phase angle is of the order 10^{-2}. The higher estimation accuracy can be obtained by using the smaller algorithm step-size.

Table 1. Estimated phasor parameters

Example	V_a	V_b	V_c	φ_c	φ_c
1	220.00	220.01	219.99	120.00	119.99
2	220.01	169.91	180.02	129.92	160.05

5. CONCLUSION

A new technique for frequency and phase estimation in three-phase power system is proposed. The proposed solution consists of three adaptive algorithms. The first adaptive algorithm finds the optimal coefficients for transformation of three-phase power system into a complex exponential. Based on the estimated coefficients, the CLMS algorithm estimates the system frequency. The third adaptive algorithm, combined with the Newton LMS, is used for estimation of phasor amplitudes and phase angles.

The simulation results confirm that the proposed algorithm in unbalanced conditions exhibits the better performances compared to the considered algorithms. In balanced three-phase systems, the proposed algorithm has similar performances to the CLMS algorithm and converges faster compared to the WLCLMS algorithm.

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