

# PERFORMANCE ANALYSIS OF CAPON AND CAPON-LIKE ALGORITHM FOR SMART ANTENNA SYSTEM

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**Abstract:** This paper presents a comparative performance study of Capon and Capon-like algorithms used in linear antenna array and his implementation on planar antenna array. We studied the effect of antenna array parameters in terms of number and element spacing and number of iterations. In simulations we studied the influence of angle of arrival on algorithm behavior as well as assessment of the optimal parameters of the array.

**Goal of the simulation is to find the best algorithm for very close signals and angles of arrival very close to zero.**

## 1. INTRODUCTION

A smart antenna system combines multiple antenna elements with a signal processor to optimize its radiation pattern automatically in response to the signal environment. Direction of Arrival (DOA) estimation and Beamforming are one of the key technology in smart antenna systems.

The goal of direction of arrival estimation is to estimate the direction of the signals from the desired users as well as the direction of interference signals. The results of DOA are then used by to adjust the weights of the adaptive beamformer so that the radiated power is maximized towards the desired users, and radiation nulls are placed in the direction of interference signals. Hence, a successful design of an adaptive array depends highly on the choice of the DOA estimation algorithm which should be highly accurate and robust. Different beamforming algorithm are used to generate the beam in the desired direction after the DOA estimation [1] [2].

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Accurate estimation of a signal direction of arrival (DOA) has received a considerable attention in communication and radar systems of commercial and military applications [3][4]. The DOA estimation using a fixed antenna has many limitations. Antenna mainlobe beamwidth is inversely proportional to its physical size. Instead of using single antenna, an array antenna system with innovative signal processing can enhance the resolution of a DOA algorithm. A sensor array has better performance than the single sensor in signal reception and parameter estimation.

In this paper we will discuss the application of estimating the DOA of multiple signals using uniform linear and planar array antenna with Capon and Capon like algorithm. The goal is to determine which algorithm gives the best results for close and small angles and shortest computational time [5] [6].

## 2. THEORY OF SMART ANTENNA SYSTEMS

Let a uniform planar array be composed of  $N_x \times N_y$  sensors arranged in the  $x$ - $y$  plane with an inter-element spacing  $d_x$  and  $d_y$ , as shown in Fig. 1, and let it receive  $M$  narrowband source signals  $s_m(t)$  from desired users arriving at directions  $\theta_1, \theta_2, \dots, \theta_M$  and  $\Phi_1, \Phi_2, \dots, \Phi_M$ . The array also receives  $I$  narrowband source signals  $s_i(t)$  from undesired (or interference) users arriving at directions  $\alpha_1, \alpha_2, \dots, \alpha_I$  and  $\beta_1, \beta_2, \dots, \beta_I$ . At a particular instant of time  $t=t_1, t_2, \dots, K$  where  $K$  is the total number of snapshots taken.

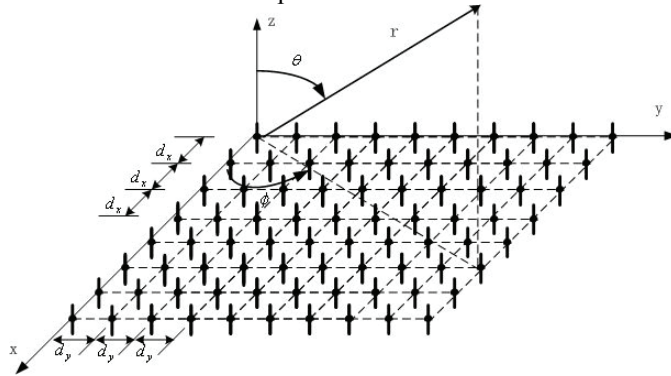


Figure 1. Rectangular planar array.

The sum of a desired signal  $x_M(t)$  from every row of antenna array can be defined as product of array factor and signal received on each antenna element

$$x_x(t) = \sum_{i=1}^{N_x} s(t) e^{j(i-1)\psi_x} = AF_x(\theta, \Phi) s(t) \quad (1)$$

and array factor for every row is given by

$$AF_x(\theta, \Phi) = \sum_{i=1}^{N_x} e^{j(i-1)\psi_x} \quad (2)$$

We can also define the array factor for every column of antenna array as

$$AF_Y(\theta, \Phi) = \sum_{k=1}^{N_y} e^{j(k-1)\psi_y} \quad (3)$$

The array factor for this type of planar array can be written as [7]

$$AF(\theta, \Phi) = \sum_{i=1}^{N_x} \sum_{k=1}^{N_y} e^{j(i-1)\psi_x} e^{j(k-1)\psi_y} \quad (4)$$

where the angles  $\psi_x$  and  $\psi_y$  represent the electrical phase shift from element to element along the array and are defined as

$$\psi_x = \frac{2\pi d_x}{\lambda} \sin(\theta) \cos(\Phi) \quad (5)$$

$$\psi_y = \frac{2\pi d_y}{\lambda} \sin(\theta) \sin(\Phi) \quad (6)$$

where  $d$  is the inter-element spacing and  $\lambda$  is the wavelength of the signal. From (5) and (6) we can see that array factor is the function of elevation ( $\theta$ ) and azimuth angle ( $\Phi$ ).

Looking at this from the viewpoint of mathematical model of a discrete signal, the desired users signal  $x_m(t)$  can be written as

$$x_m(t) = A_m(\theta, \Phi) s_m(t) \quad (7)$$

where  $A_m(\theta, \Phi)$  is the  $N \times M$  matrix of the desired users signal direction vectors (discrete form of array factor  $AF_x(\theta, \Phi)$ ) and is given by [2]

$$A_m(\theta, \Phi) = [a(\theta_1, \Phi_1), a(\theta_2, \Phi_2), \dots, a(\theta_M, \Phi_M)] \quad (8)$$

where  $N$  is the number of array elements and equal to  $N = N_x \times N_y$  and  $s_m(t)$  is the  $M \times 1$  desired users source waveform vector defined as

$$s_m(t) = [s_1(t) \quad s_2(t) \quad \dots \quad s_M(t)]^T \quad (9)$$

where  $T$  is Transposition operator. We can also define the undesired users signal  $x_i(t)$  vector

$$x_i(t) = A_i(\theta, \Phi) s_i(t) \quad (10)$$

where  $A_i$  is the  $N \times I$  matrix of the undesired users signal direction vectors and  $s_i(t)$  is the  $I \times 1$  undesired users source waveform vector.

The overall received signal vector  $x(t)$  is given by the superposition of the desired users signal vector, undesired users signal vector and  $N \times 1$  vector  $n(t)$  which represents a white Gaussian noise on antenna elements. Hence,  $x(t)$  can be written as

$$x(t) = x_m(t) + n(t) + x_i(t) \quad (11)$$

If the algorithm is applied on uniform linear array,  $\phi$  is fixed or equal to zero.

### A. CAPON algorithm

Capon algorithm is a conventional spectral-based method, introduced by J. Capon, used to improve the resolution of Bartlett algorithm [8]. The main idea of Capon algorithm is to minimize the received power of the incoming signal in all direction while maintaining a unity gain in ‘look direction’. The condition set by this algorithm is given as:

$$w^H a(\theta, \phi) = 1, \quad (12)$$

where  $w$  represents weight coefficients.

Applying Lagrange optimization method the optimized weight coefficients are given by:

$$w_{Capon} = \frac{\hat{R}^{-1} a(\theta, \phi)}{a^H(\theta, \phi) \hat{R}^{-1} a(\theta, \phi)} \quad (13)$$

where  $a(\theta, \phi)$  represents steering vector and  $\hat{R}$  covariance matrix.

Using the optimized weight, the spatial power spectrum of Capon is given as:

$$P_{Capon} = \frac{1}{a^H(\theta, \phi) \hat{R}^{-1} a(\theta, \phi)} \quad (14)$$

### B. „CAPON-like” algorithm

Capon-like algorithm is optimized for directional antenna arrays and it’s based on Capon algorithm [9]. Steering vector in this case can be expressed as

$$A[\theta, \phi] = [g(\theta_1, \phi_1) a(\theta_1, \phi_1), \dots, g(\theta_N, \phi_N) a(\theta_N, \phi_N)]^T \quad (15)$$

where  $g(\theta_i, \phi_i)$  is the array gain in a specific direction defined with angles  $\theta$  and  $\phi$ . For antenna arrays with isotropic elements  $g(\theta_i, \phi_i) = 1$  for  $i=1, 2, \dots, N$ . However, in the case of antenna arrays with directional elements,  $g(\theta_i, \phi_i)$  is determined from the array gain pattern and thus significantly affect the values of  $a(\theta_i, \phi_i)$ . Since the array response is affected by the gain pattern in a ‘look direction’, the constraint in Capon-like algorithm can be given by:

$$w^H a(\theta, \phi) = g(\theta, \phi) \quad (16)$$

Applying Lagrange optimization technique weight coefficients can be given by:

$$w = \frac{\hat{R}^{-1} a(\theta, \phi) g^H(\theta, \phi)}{a^H(\theta, \phi) \hat{R}^{-1} a(\theta, \phi)}. \quad (17)$$

Power spectrum of the Capon-like algorithm is given as:

$$P = \frac{a^H(\theta, \phi) a(\theta, \phi)}{a^H(\theta, \phi) \hat{R}^{-1} a(\theta, \phi)} \quad (18)$$

If the Capon like algorithm is applied on linear antenna array we can write  $a(\theta, \phi) = a(\theta)$  and  $g(\theta, \phi) = g(\theta)$ .

### 3. SIMULATION AND PERFORMANCE EVALUATION

Through simulations we studied the effect of antenna array parameters in terms of its size and element spacing, and also the influence of angle of arrival. We used signal defined by equation (11) in all simulations.

Fig. 2 shows comparative simulation result diagram of array factor when signals arrive from  $\theta_1 = -30^\circ$ ,  $\theta_2 = 5^\circ$ ,  $\theta_3 = 20^\circ$  and  $\theta_4 = 70^\circ$  for both Classical DOA and Capon like algorithm. Simulation is done for uniform linear array with 10 elements.

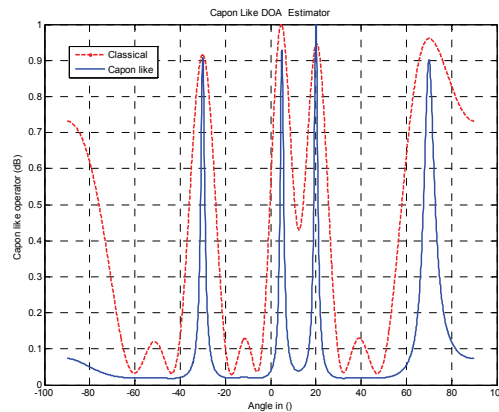


Figure 2. Comparative simulation result diagram of array factor for Classical and Capon like algorithm when signals arrive from  $\theta_1 = -30^\circ$ ,  $\theta_2 = 5^\circ$ ,  $\theta_3 = 20^\circ$  and  $\theta_4 = 70^\circ$ .

Table 1. Comparative results of DOA estimation for Classical, Capon and Capon like algorithm.

Algorithm	$\theta_1 = -30^\circ$	$\theta_2 = 5^\circ$	$\theta_3 = 20^\circ$	$\theta_4 = 70^\circ$
Classical	$-30.1^\circ$	$4.7^\circ$	$20.5^\circ$	$70.4^\circ$
Capon	$-29.9^\circ$	$5.1^\circ$	$20.2^\circ$	$70.2^\circ$
Capon like	$-29.9^\circ$	$5.1^\circ$	$20.1^\circ$	$70.2^\circ$

From Fig. 2 and Table 1 we can see that Capon like algorithm achieves very good resolution of angle estimation and power dissipation in unwanted directions.

Computational time is calculated, and for Classical DOA algorithm required time is 0.645s. Time needed for Capon algorithm is 0.174s, and for Capon like 0.110s. We can see that time for calculating Capon like algorithm is six times shorter than time required for Classical DOA and 37% shorter than time required for Capon algorithm.

Short computational time is the key advantage of Capon like algorithm.

Fig. 3 shows simulation result diagram of antenna array factor for Capon like algorithm when signals arrive from  $\theta_1 = -24^\circ$ ,  $\theta_2 = 13^\circ$ ,  $\theta_3 = 21^\circ$  and  $\theta_4 = 70^\circ$  for different number of algorithm iterations.

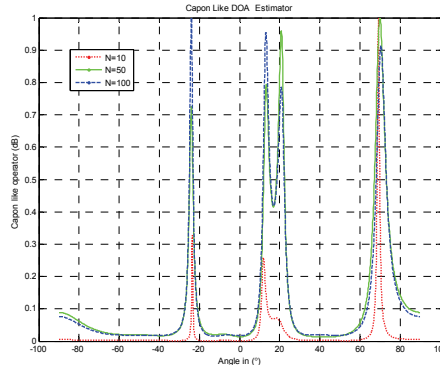


Figure 3. Simulation result diagram of antenna array factor for Capon like algorithm when signals arrive from  $\theta_1 = -24^\circ$ ,  $\theta_2 = 13^\circ$ ,  $\theta_3 = 21^\circ$  and  $\theta_4 = 70^\circ$  for different number of iterations

Table 2. Comparative results of DOA estimation for Classical, Capon and Capon like algorithm for different number of algorithm iterations.

Algorithm	Classical			Capon			Capon like		
No of samples	10	50	100	10	50	100	10	50	100
$\theta_1 = -24^\circ$	-24.1°	-23.8°	-23.7°	-24.4°	-24.1°	-23.8°	-24.1°	-24°	-24°
$\theta_2 = 13^\circ$	14.7°	16.2°	16.9°	10.8°	13.5°	13.3°	14.3°	13.5°	13.3°
$\theta_3 = 21^\circ$	Not found	Not found	Not found	22.4°	20.7°	21.2°	18.8°	21°	21.1°
$\theta_4 = 70^\circ$	70.7°	69.1°	70.1°	Not found	70°	70.2°	68.2°	70°	70.3°

From Fig. 3 and Table 2 we can see that Capon like algorithm achieves good results for small number of iteration, only 50. For 10 iteration, estimation of three of the four angles is possible, which is very good and it's an advantage of Capon like algorithm compared to other DOA algorithms.

Fig. 4 shows simulation result diagram of antenna array factor for Capon like algorithm when signals arrive from  $\theta_1 = -24^\circ$ ,  $\theta_2 = 13^\circ$ ,  $\theta_3 = 21^\circ$  and  $\theta_4 = 70^\circ$  for different number of array elements.

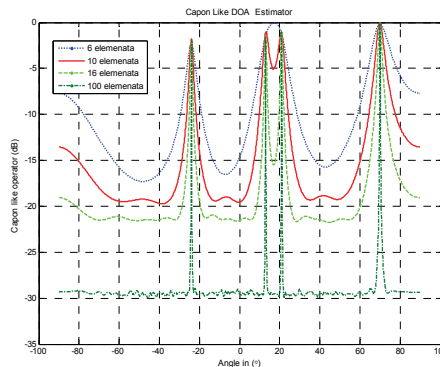


Figure 4. Simulation result diagram of antenna array factor for Capon like algorithm when signals arrive from  $\theta_1 = -24^\circ$ ,  $\theta_2 = 13^\circ$ ,  $\theta_3 = 21^\circ$  and  $\theta_4 = 70^\circ$  for different number of array elements.

From the Fig. 4 we can see that when the number of array elements increased, the performance of the system improves.

Fig. 5 shows simulation result diagram of antenna array factor for Capon like algorithm when signals arrive from  $\theta_1 = -24^\circ$ ,  $\theta_2 = 13^\circ$ ,  $\theta_3 = 21^\circ$  and  $\theta_4 = 70^\circ$  for different element spacing ( $d = \lambda$ ,  $d = 1/2 \lambda$ ,  $d = 2/3 \lambda$  and  $d = 1/8 \lambda$ ).

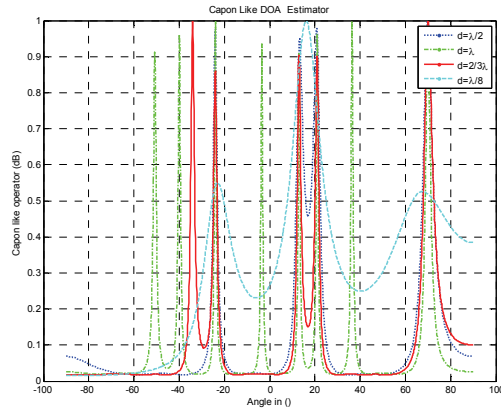


Figure 5. Simulation result diagram of antenna array factor for Capon like algorithm when signals arrives from  $\theta_1 = -24^\circ$ ,  $\theta_2 = 13^\circ$ ,  $\theta_3 = 21^\circ$  and  $\theta_4 = 70^\circ$  for different element spacing ( $d = \lambda$ ,  $d = 1/2 \lambda$ ,  $d = 2/3 \lambda$  and  $d = 1/8 \lambda$ ).

From Fig. 5 we can see that Capon like algorithm shows a sensitivity to element spacing. Only for spacing equals  $\lambda/2$  algorithm works, for other combination of spacing, the algorithm estimates non-existent angle of arrival.

Fig. 6 shows comparative simulation result diagram of array factor for Capon and Capon like algorithm when signals arrive from  $\theta_1 = -30^\circ$ ,  $\theta_2 = -40^\circ$  and  $\theta_4 = 50^\circ$ .

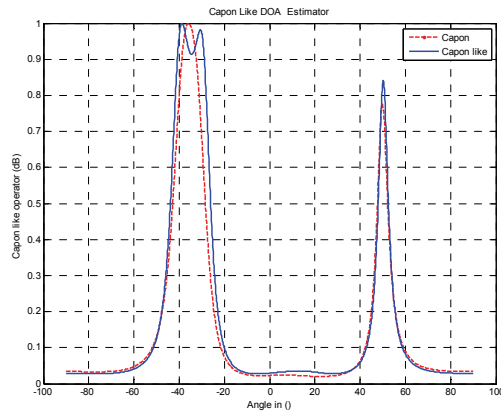


Figure 6. Comparative simulation result diagram of array factor for Capon and Capon like algorithm when signals arrive from  $\theta_1 = -30^\circ$ ,  $\theta_2 = -40^\circ$  and  $\theta_4 = 50^\circ$ .

Table 3. Comparative results of DOA estimation for Capon and Capon like algorithm.

Algorithm	$\theta_1 = -40^\circ$	$\theta_2 = -30^\circ$	$\theta_4 = 50^\circ$
Capon	$-35.9^\circ$	Not found	$50.2^\circ$
Capon like	$-38.8^\circ$	$-30.6^\circ$	$50.2^\circ$

From Fig. 6 and Table 3 we can see that Capon like algorithm achieves better results when signals arrive from very close angles. Capon algorithm was not able to assess the two angles ( $\theta_1 = -40^\circ$  and  $\theta_2 = -30^\circ$ ). This behavior of the Capon like algorithm is the same as in the case of very small and close angles of arrival (close to  $0^\circ$  or  $90^\circ$ ), which is important for spatial filtering in modern communications.

Fig. 7 shows comparative simulation result diagram of array factor for Capon and Capon like algorithm when signals arrive from  $\theta_1 = -50^\circ$ ,  $\theta_2 = 20^\circ$  and  $\theta_3 = 50^\circ$  for 10 iterations.

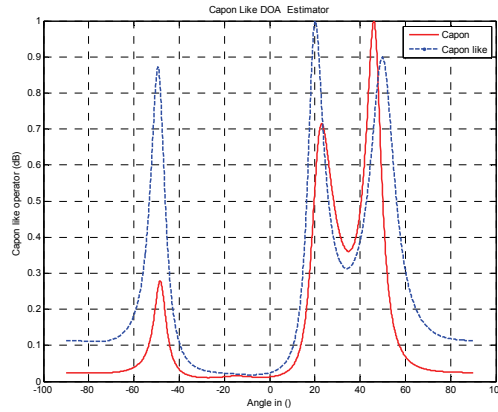


Figure 7. Comparative simulation result diagram of array factor for Capon and Capon like algorithm when signals arrive from  $\theta_1 = -50^\circ$ ,  $\theta_2 = 20^\circ$  and  $\theta_3 = 50^\circ$  for 10 iterations.

Table 4. Comparative results of DOA estimation for Capon and Capon like algorithm.

Algorithm	$\theta_1 = -50^\circ$	$\theta_2 = 20^\circ$	$\theta_4 = 50^\circ$
Capon	$-47.3^\circ$	$24.1^\circ$	$46.4^\circ$
Capon like	$-50.05^\circ$	$20.1^\circ$	$50^\circ$

From Fig. 7 and Table 4 we can see that for small number of iterations, which is the current trend in the development of the adaptive algorithm, exact angle estimation is achieved by using Capon like algorithm, while the Capon algorithm is still calculating the angles with a significant error.

For further analysis of the performance of this algorithm, the implementation of Capon like algorithm is done with planar antenna arrays. Fig. 8 shows the simulation result diagram of array factor for Capon like algorithm applied on planar antenna array when signals arrive from  $\theta_1 = 20^\circ, \phi_1 = 80^\circ$ ,  $\theta_2 = 30^\circ, \phi_2 = 40^\circ$  and  $\theta_3 = 60^\circ, \phi_3 = 60^\circ$  for different number of iterations.



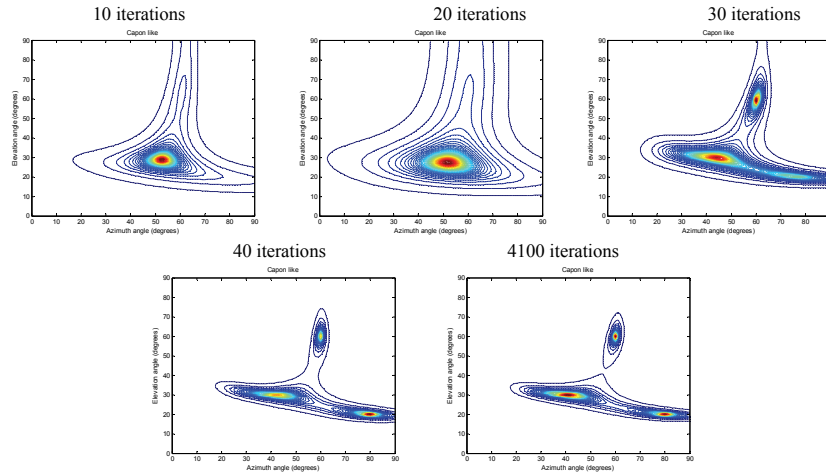


Figure 8. Simulation result diagram of array factor for Capon like algorithm applied on planar antenna array when signals arrives from  $\theta_1=20^\circ$ ,  $\phi_1=80^\circ$ ,  $\theta_2=30^\circ$ ,  $\phi_2=40^\circ$  and  $\theta_3=60^\circ$ ,  $\phi_3=60^\circ$  for different number of iterations.

Based on Fig. 8 we can conclude that Capon like algorithm applied on planar antenna arrays estimates arriving signals very precisely after only 40 iterations with the same error as after 4100 iterations.

Fig. 9 shows simulation result diagram of array factor for Capon like algorithm applied on planar antenna array when signals arrive from very close angles  $\theta_1=30^\circ$ ,  $\phi_1=60^\circ$ ,  $\theta_2=40^\circ$ ,  $\phi_2=60^\circ$  and  $\theta_3=50^\circ$ ,  $\phi_3=50^\circ$ .

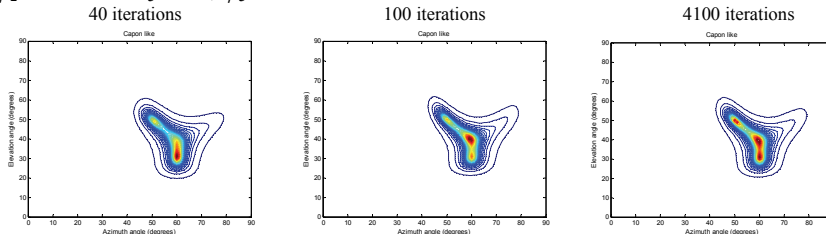


Figure 9. Simulation result diagram of array factor for Capon like algorithm applied on planar antenna array when signals arrives from very close angles  $\theta_1=30^\circ$ ,  $\phi_1=60^\circ$ ,  $\theta_2=40^\circ$ ,  $\phi_2=60^\circ$  and  $\theta_3=50^\circ$ ,  $\phi_3=50^\circ$ .

From Fig. 9 we can see that in case of very close angles, the Capon like algorithm need more time and more number of iterations (minimum 40) to achieve satisfying error.

#### 4. CONCLUSION

In this paper, the performance of two DOA algorithms Capon and Capon like are discussed and compared. The algorithms are also implemented on planar antenna arrays. These algorithms are used in smart antenna systems to enhance mobile communication performance.

Simulations were aimed at determining the speed of calculation of improved Capon-like algorithm as compared to Capon algorithm and its accuracy in the determination of the very close angles of arrival.

In this paper, the effect of the number of antenna elements in the behavior of the algorithm, is analyzed and has been found that Capon-like algorithm gives better results in the case of arrays with a small number of elements. In the case of very close angles and angles very close to  $0^\circ$ , Capon like algorithm, for only 40 iterations, achieves very good results.

As for the number of iterations, Capon-like algorithm achieves better results, and only 10 iterations are enough to make estimation of incident angles. Computing speed, a small number of elements and a small number of iterations are the main advantages of Capon-like algorithm compared with other DOA algorithms.

In the case of large planar arrays and a large number of iterations, the simulation results are almost the same as in the case of Capon algorithm except for the speed estimation which is less.

Simulations for close and small angles of arrival is shown that Capon-like algorithm achieves better results, which is the tendency of today's development of algorithms.

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