FLATNESS BASED CONTROL OF AUTOMATED GUIDED VEHICLE

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Abstract: This paper presents simulation of the motion of autonomous vehicle using the flatness based control. Based on the kinematic model of the vehicle motion a feed-forward trajectory tracking controller has been designed. For the system stabilization in the presence of disturbances a feedback controller based on linear error dynamics has been constructed. In either case, comparative study between real moving and desired one has been presented.

1. INTRODUCTION

The trajectory planning, formation maneuver and motion control, in particular parking control problems for mobile robots and other autonomous ground vehicles have received considerable attention over the last two decades. Various intelligent control strategies, such as fuzzy logic and neural networks-based control were adopted by many researchers [1], [2], [3]. The last generation of automated guided vehicles (AGV) systems incorporates wireless guidance systems, using laser or inertial sensors that permit to locate these vehicles with the precision needed for performing accurate maneuvers [4]. However, popular commercial industrial navigation systems, namely laser navigation ones, induce problems related to the low pose estimation rate. This problem is usually solved by fusing those position estimations with odometry information, whether from wheel encoders or from inertial sensors. Nevertheless, the position estimation rate is still relatively low, which represents a key point that the tracking technique should take into account.

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The typical load transfer operations of AGV systems consist of pick-up and delivery services between pick-up/delivery stations. In order to do so, the AGV should navigate between stations performing precise docking maneuvers to pick-up or deliver the transport load. Navigation between stations is referred as point-to-point navigation whereas precise maneuvers are referred as docking operations. The docking maneuvers need higher accuracy than navigation tasks because the vehicle can collide against the station/load or knock down the load due to an inaccurate maneuver. Hence, slight errors in docking maneuvers can damage the load, the station, and the vehicle. Moreover, the docking operations should be performed as fast as possible avoiding vibrations induced by eventual high variations in control actions. These vibrations result critical for both the accuracy of the docking operation and for the stability of the load.

One of the classes of systems for which trajectory planning is particularly easy are so-called differentially flat systems. Roughly speaking, a system is differentially flat if we can find a set of outputs such that all states and inputs can be determined from these outputs without integration. Differentially flat systems were originally studied by M. Fliess [5]. Flatness-based control techniques have been developed and applied in many industrial processing with a great success in solving planning and tracking problems of reference trajectories such as thermal process control [6], motors control [7], chemical reactor control [8], crane control [9] etc.

In this paper we concetrate on characterizing differential flatness for automated guided vehicle. The flatness property of the kinematic model simplifies the trajectory planning and motion control of the car-like autonomous vehicle. The two approaches are adopted: feed-forward and dynamic feedback control.

2. FLAT SYSTEMS

A system is called flat if there exists a set of differentially independent variables which can be used to calculate all other system variables using differentiation but no integration, i.e. without solving differential equations [5]. More precisely, if the system has states $x \in R^n$ and inputs $u \in R^m$ then the system is flat if we can find outputs $y \in R^m$ such that:

$$x = \psi_x(y, \dot{y}, ..., y^{(\delta)}) \tag{1}$$

$$u = \psi_u(y, \dot{y}, ..., y^{(\gamma)}) \, . \tag{2}$$

Flat system admits a free differential parametrization, with the flat output y playing the role of the functional parameter. Due to the differential independence of the flat output y, reference trajectories $t \to y_r(t)$ for its components can be chosen independently and freely. Compatible trajectories of all other system variables can then be expressed as functions of y_r and its derivatives. The only restriction the reference trajectories of the flat output must satisfy is that they must be smooth enough (i.e. sufficiently differentiable) so that all time derivatives required in the calculation are well defined.

Parametrizing a system in terms of a flat output simplifies the design of a feed-forward control allowing us to steer the system along a reference trajectory. Indeed, it is sufficient to carefully choose a reference trajectory $y_r(t)$ for the flat output and use it in the equation of the control input (2) which immediately yields the control trajectory with:

$$u_r(t) = \psi_u(y_r(t), \dot{y}_r(t), ..., y_r^{(\gamma)}(t)) . \tag{3}$$

As it can be seen from(3), in order to get a continuous control trajectory u_r, the components $y_{r,i}$ of the trajectory y_r must be continuously differentiable at least γ_i times.

An important class of trajectory tracking control problems results from start-up and shut-down, which are often characterized by rest-to-rest transition. For instance, docking operations of the AGV are such a kind of transitions. In general, in the neighborhood of different operating points the system behavior will be considerably different. As a consequence, using a approximated model based on local linearization for the control design is often insufficient in these cases, and nonlinear flatness based control shows great advantages.

If we use a flatness based parametrization of the initial and terminal operating points in transition, these points are characterized by different constant values of y at starting time $t = 0$ and at the final time $t = t_f$. At these equilibrium points all derivatives are equal to zero.

Any sufficiently smooth function can be used to define the reference trajectory $y_r(t)$. The calculations are particularly simple if we use polynomials. Then, the parameters of the polynomials follow from the initial and final values of y_r by solving a linear system of equations. If, as an example, we need derivatives of y up to order 1 to calculate u, and if u is required to be continuous both at $t = 0$ and $t = t_f$, then we have 4 conditions ($y(0) = y_0$, $y(t_f) = y_f$, $\dot{y}(0) = 0$, $\dot{y}(t_f) = 0$) and we need at least a polynomial of order 3:

$$y(t) = c_0 + c_1 t + c_2 t^2 + c_3 t^3 \,. \tag{4}$$

The unknown coefficients $c_0, c_1, c_{2,}, c_3$ from (4) follow from the linear system of equations:

$$\begin{aligned} y_0 &= c_0 + c_1 \cdot 0 + c_2 \cdot 0 + c_3 \cdot 0 \\ y_f &= c_0 + c_1 \cdot t_f + c_2 \cdot t_f^2 + c_3 \cdot t_f^3 \\ 0 &= c_1 + 2c_2 \cdot 0 + 3c_3 \cdot 0 \\ 0 &= c_1 + 2c_2 \cdot t_f + 3c_3 \cdot t_f^2 \,. \end{aligned} \tag{5}$$

Using the (3) to lead the system along reference trajectory by feed-forward control would require that the model is an exact description of reality, that there are no exterior perturbations, and that the initial conditions used to calculate the control are those of the real car. Therefore, we should add feedback to our design. We do this by designing a control law allowing us to asymptotically bring the system to the reference trajectory in case there is an initial error. This may be considered as a stabilizing trajectory tracking controller.

The variable used to define the tracking behavior is the vector pointing from the reference to the actual trajectory of the flat output components, the so called tracking error:

$$e_i = y_i - y_{i,r} \,. \tag{6}$$

Depending on the order of derivatives of the components y_i which appears in (3), tracking error can be described with linear differential equation:

$$e_i^{\gamma_i} + k_{\gamma-1} e_i^{\gamma_i - 1} + \ldots + k_0 e_i = 0 \,, \tag{7}$$

Our control design is, thus, based on the linear error dynamics we have chosen. Based on our knowledge of stability properties of linear systems, or physical insight, we have achieved asymptotically stable tracking of the reference trajectory. The convergence of the trajectory of the flat output implies convergence of the trajectory of all other variables to their respective references.

3. FLATNESS CONTROL DESIGN

The kinematic model of a vehicle is depicted in Fig 1. As we can see, vehicle moves along the path described with the curve C. The point $Y=(y_1,y_2)$ is fixed to the rear axle center and it follows the path. Moreover, orientation of the vehicle is determined by the angle θ. Control actions are performed by the velocity v and steering angle φ. Finally, let the constant parameter l denote the distance between front and rear axle mid-points.

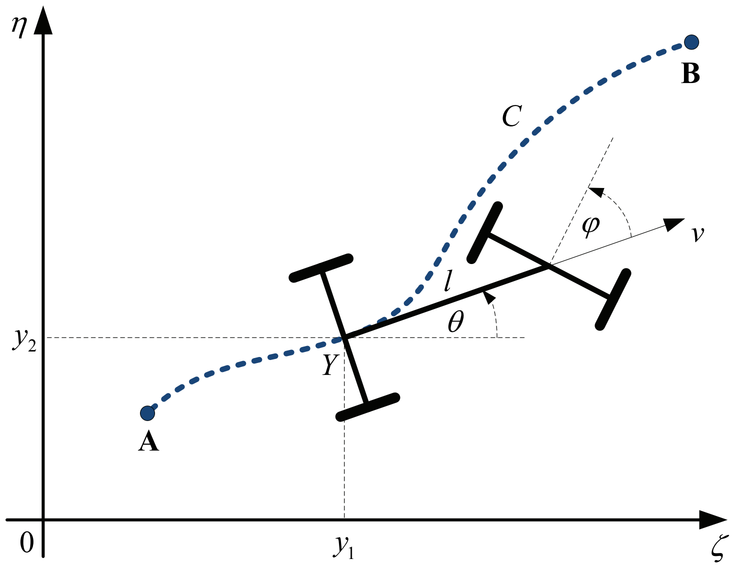

Fig 1. The vehicle in a reference frame fixed in the plane

We consider the kinematic model of the vehicle:

$$\dot{y}_1 = v\cos\theta \tag{8}$$

$$\dot{y}_2 = v\sin\theta \tag{9}$$

$$\dot{\theta} = v\frac{\tan\varphi}{l}. \tag{10}$$

On the basis of this model it can be shown that the knowledge of the trajectory of point Y is sufficient to determine the motion of the vehicle. Indeed, all the variables can be expressed as functions of (y_1, y_2) and its derivatives:

$$\theta = \arctan\left(\frac{\dot{y}_2}{\dot{y}_1}\right) \tag{11}$$

$$v = \pm\sqrt{\dot{y}_1^2 + \dot{y}_2^2} \tag{12}$$

$$\varphi = \arctan\left(\frac{l\dot{\theta}}{v}\right) = \pm\arctan\left(\frac{l(\ddot{y}_2\dot{y}_1 - \ddot{y}_1\dot{y}_2)}{(\dot{y}_1^2 + \dot{y}_2^2)^{\frac{3}{2}}}\right). \tag{13}$$

This parametrization shows that the vehicle is a flat system and coordinates (y_1, y_2) of the rear axle center point represent a flat output for this system.

Choosing the variables v and φ for control inputs, a feed-forward control for the vehicle can be designed. It is sufficient to choose a reference trajectory for the flat output $(y_{1,r}(t), y_{2,r}(t))$ and use it in (11)-(13). In order to simplify this, first we choose $y_{1,r}(t)$. Then, we may define an explicit relation between the two coordinates $y_{2,r} = f(y_{1,r})$. Additionally, we simplify this by using polynomials for these functions. With this choice, in order to determine the order of polynomials, after some calculations, we rewrite (12) and (13) as:

$$v = \pm\dot{y}_1\sqrt{1+\left(\frac{df}{dy_1}\right)^2} \tag{14}$$

$$\frac{\tan\varphi}{l} = \frac{\dfrac{d^2 f}{dy_1^2}}{\left(1+\left(\dfrac{df}{dy_1}\right)^2\right)^{3/2}}. \tag{15}$$

As it can be seen from (14) and (15), reference trajectories for the flat output follow from path parametrization $y_2 = f(y_1)$ and time parametrization $y_1(t)$. Function $f(y_1)$ must be at least twice differentiable in order to find steering angle control from (15). Also, it must include initial $(\theta_A, \varphi_A, y_{2,A})$ and final position $(\theta_B, \varphi_B, y_{2,B})$ of the vehicle at the points $y_{1,A}$ and $y_{1,B}$, so we need a polynomial of degree 5:

$$y_2 = f(y_1) = c_0 + c_1 y_1 + c_2 y_1^2 + c_3 y_1^3 + c_4 y_1^4 + c_5 y_1^5 . \tag{16}$$

The 6 unknown coefficients are directly calculated by solving linear system of equations (similar to what is done in (5)) with known 6 conditions:

$$\begin{aligned}
&y_2(y_{1,A}) = y_{2,A} \\
&y_2'(y_{1,A}) = \frac{df}{dy_1}(y_{1,A}) = \tan\theta_A \\
&y_2''(y_{1,A}) = \frac{d^2 f}{dy_1^2}(y_{1,A}) = \frac{\tan\varphi_A}{l}\left(1+\tan^2\theta_A\right)^{3/2} \\
&y_2(y_{1,B}) = y_{2,B} \\
&y_2'(y_{1,B}) = \frac{df}{dy_1}(y_{1,B}) = \tan\theta_B \\
&y_2''(y_{1,B}) = \frac{d^2 f}{dy_1^2}(y_{1,B}) = \frac{\tan\varphi_B}{l}\left(1+\tan^2\theta_B\right)^{3/2} .
\end{aligned} \tag{17}$$

From (14) we can see that $y_1(t)$ appears as first order derivative in velocity trajectory, so we need a polynomial of degree 3:

$$y_1(t) = d_0 + d_1 t + d_2 t^2 + d_3 t^3 . \tag{18}$$

The coefficients from (18) can be derived from two initial conditions at the time $t = 0$ and two final conditions at the time $t = t_f$:

$$\begin{aligned}
&y_1(0) = y_{1,A} \\
&\dot{y}_1(0) = 0 \\
&y_1(t_f) = y_{1,B} \\
&\dot{y}_1(t_f) = 0.
\end{aligned} \tag{19}$$

As we know, feed-forward steering alone will often not be sufficient to achieve satisfactory tracking along a reference trajectory. This implies that we must introduce appropriate feedback to make sure that the actual trajectory stay near the planned one, even if the actual initial conditions do not coincide with those used for planning, and if the model does not match the actual behavior very well. The tracking error yields:

$$e(t) = (y_1(t) - y_{1,r}(t), y_2(t) - y_{2,r}(t)), \tag{20}$$

and we chose desired error dynamics as:

$$\begin{bmatrix} \ddot{e}_1 \\ \ddot{e}_2 \end{bmatrix} + K_1 \begin{bmatrix} \dot{e}_1 \\ \dot{e}_2 \end{bmatrix} + K_0 \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} = 0 . \tag{21}$$

Here, K_0 and K_1 can be chosen as positive scalars. In order to establish the relation between our desired error dynamics and vehicle model, from (21) we derive expression for acceleration:

$$\begin{bmatrix} \ddot{y}_1 \\ \ddot{y}_2 \end{bmatrix} = \begin{bmatrix} \ddot{y}_{1,r} \\ \ddot{y}_{2,r} \end{bmatrix} - K_1 \begin{bmatrix} v\cos\theta - \dot{y}_{1,r} \\ v\sin\theta - \dot{y}_{2,r} \end{bmatrix} - K_0 \begin{bmatrix} y_1 - y_{1,r} \\ y_2 - y_{2,r} \end{bmatrix}, \tag{22}$$

where we have used (8) and (9) to replace the actual velocities in $\dot{e} = y - \dot{y}_r$. Furthermore, by differentiating the equations of the direction velocities (8) and (9) we obtain:

$$\begin{bmatrix} \ddot{y}_1 \\ \ddot{y}_2 \end{bmatrix} = \underbrace{\begin{pmatrix} \cos\theta & -\sin\theta \\ sin\theta & \cos\theta \end{pmatrix}}_{R(\theta)} \begin{bmatrix} \dot{v} \\ v\dot{\theta} \end{bmatrix} = R(\theta) \begin{bmatrix} \dot{v} \\ \frac{v^2}{l}\tan\varphi \end{bmatrix}. \tag{23}$$

As the right hand sides of (23) and (22) must be equal:

$$\begin{bmatrix} \dot{v} \\ \frac{v^2}{l}\tan\varphi \end{bmatrix} = R^{-1}(\theta) \left(\begin{bmatrix} \ddot{y}_{1,r} \\ \ddot{y}_{2,r} \end{bmatrix} - K_1 \begin{bmatrix} v\cos\theta - \dot{y}_{1,r} \\ v\sin\theta - \dot{y}_{2,r} \end{bmatrix} - K_0 \begin{bmatrix} y_1 - y_{1,r} \\ y_2 - y_{2,r} \end{bmatrix} \right). \tag{24}$$

Equation (24) involves the position $y=(y_1,y_2)$ and the orientation θ, the known reference trajectory with $y_r(t)$, $\dot{y}_r(t)$ and $\ddot{y}_r(t)$, as well as the control inputs v and φ. In addition, the time derivative of $\dot{v}$ of one control input occurs. This means that we should consider the first row of (24) as a differential equation for v, which can be solved in the controller if the position (y_1,y_2) and the orientation θ are measured. Solving this (first order) differential equation requires an initial condition $v(0)$, which can be obtained from the reference trajectory as $v(0)=\sqrt{\dot{y}_{1,r}^2(0)+\dot{y}_{2,r}^2(0)}$. After (numerically) solving the differential equation, the value of v can be used in the second row of (24) to calculate $\tan\varphi$ and with this φ as the second control input. Thus, our dynamic feedback controller can be summarized as follows:

$$\dot{v}=\begin{bmatrix}\cos\theta & \sin\theta\end{bmatrix}\lambda(v,y_1,y_2,\theta,y_r,\dot{y}_r,\ddot{y}_r) \tag{25}$$

$$v(0)=\sqrt{\dot{y}_{1,r}^2(0)+\dot{y}_{2,r}^2(0)} \tag{26}$$

$$\varphi=\arctan\left(\frac{l}{v^2}\begin{bmatrix}-\sin\theta & \cos\theta\end{bmatrix}\lambda(v,y_1,y_2,\theta,y_r,\dot{y}_r,\ddot{y}_r)\right) \tag{27}$$

$$\lambda(v,y_1,y_2,\theta,y_r,\dot{y}_r,\ddot{y}_r)=\begin{bmatrix}\ddot{y}_{1,r}\\ \ddot{y}_{2,r}\end{bmatrix}-K_1\begin{bmatrix}v\cos\theta-\dot{y}_{1,r}\\ v\sin\theta-\dot{y}_{2,r}\end{bmatrix}-K_0\begin{bmatrix}y_1-y_{1,r}\\ y_2-y_{2,r}\end{bmatrix}. \tag{28}$$

The structure of the controlled system is depicted in Fig 2.

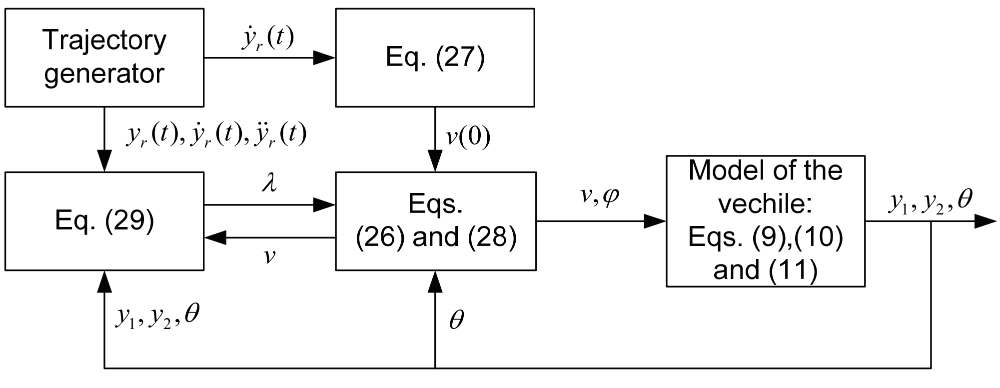

Fig 2. Structure of the tracking control loop for the vehicle model

4. SIMULATION RESULTS

Simulation for the flatness feedback control of the vehicle can be generated in MATLAB. The simulation is made for the following parameters: $(y_{1,A}, y_{2,A}, \theta_A, \varphi_A, v_A) = (0.5, 0.5, 0, 0, 0)$ at $t = 0s$ and $(y_{1,B}, y_{2,B}, \theta_B, \varphi_B, v_B) = (5, 2, 0, 0, 0)$ at $t_f = 5s$. The results obtained are shown in the following figures. Fig. 3. shows that vehicle starts moving outside of its desired (planned) position, which implies initial tracking error. Dynamic behaviour of this error is described with linear transfer function containing two poles at (-2,-2).

Stabilizing feedback controller provides convergence of the flat output $y = (y_1, y_2)$ to its reference trajectory as it is shown in Fig. 4. and 5. Consequently, all the other variables such as the orientation angle (Fig 6.), velocity (Fig. 7.) and steering angle (Fig. 8.) converge to their reference trajectories.

As can be seen in the control law (24), if the tracking error is zero, the control inputs velocity and steering angle are the same as the control inputs obtained from feed-forward control as a function of the flat output reference and its derivatives.

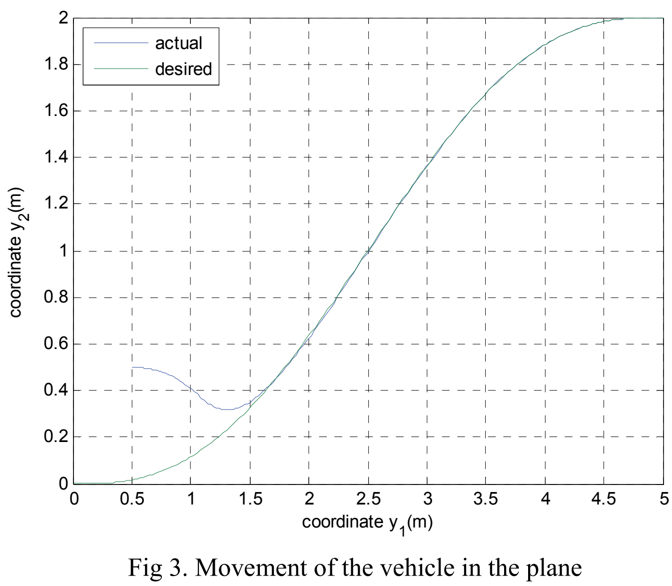

Fig 3. Movement of the vehicle in the plane

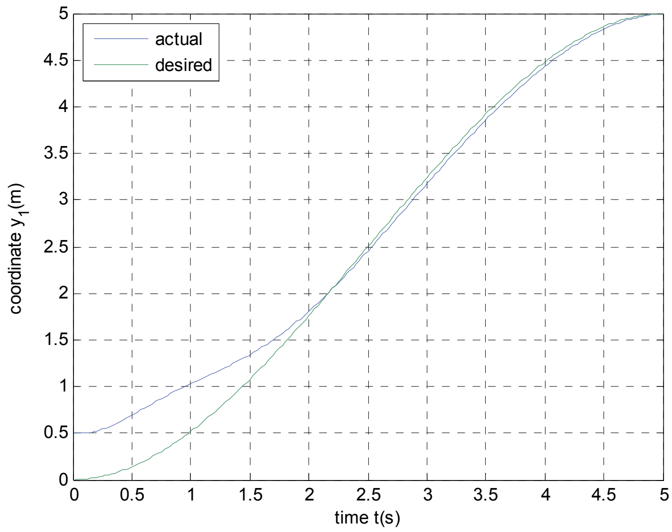

Fig 4. Trajectory of coordinate $y_1(t)$

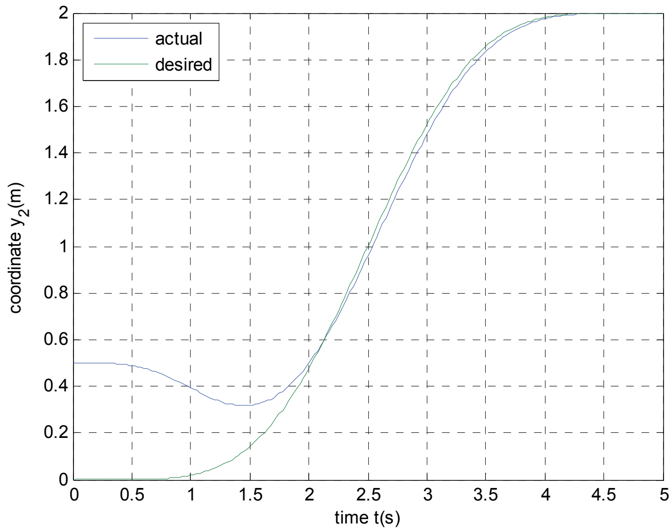

Fig 5. Trajectory of coordinate $y_2(t)$

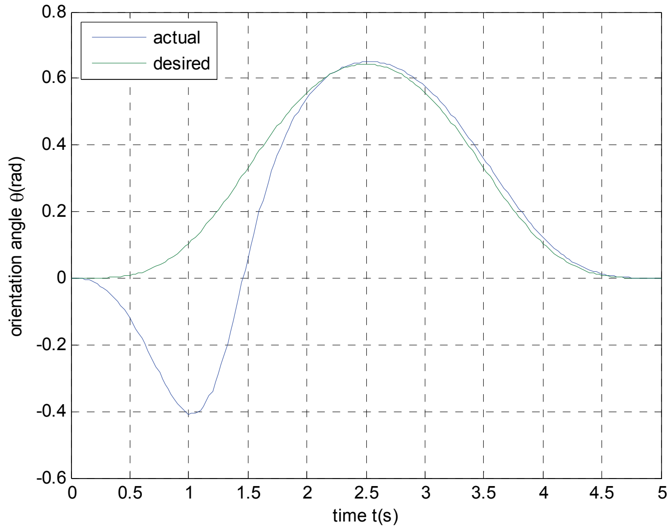

Fig 6. Trajectory of orientation angle $\theta(t)$

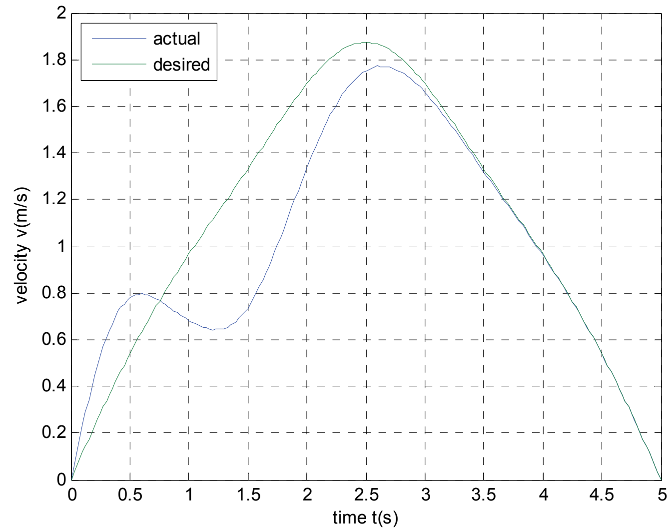

Fig 7. Trajectory of velocity $v(t)$

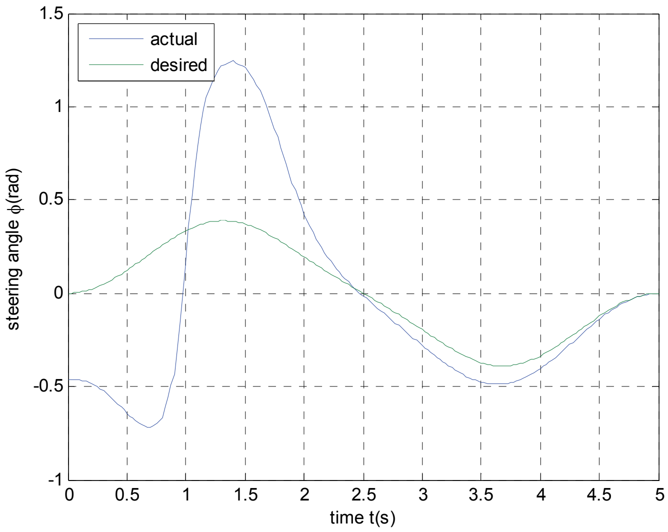

Fig 8. Trajectory of steering angle $\varphi(t)$

5. CONCLUSION

Fully automatic driving of personal cars is a far reaching challenge for the automotive industry. Many control strategies are being developed to accomplish this goal. This paper is devoted to the tracking control problem for a kinematic model of the automated guided vehicle. We obtain a simple solution thanks to the flatness property of the vehicle. The flatness based control is used to generate the desired trajectory and to force the vehicle to follow it. Simulation results show the proposed algorithm efficiencies.

Our design approach splits into two phases. First, we choose an appropriate reference trajectory such as to nominally achieve a desired motion. Hence, we obtain the corresponding control trajectories that we may use to steer the system along those trajectories. Second, we appropriately choose a dynamics for the error on the flat output trajectory and calculate the corresponding law based on the stabilizing feedback control. This two step design can be applied to all flat systems. The proposed method is simple and efficient and has made its way into industrial practice.

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